

## **Étude expérimentale sur la résistance externe optimale d'un générateur thermoélectrique**

### **Experimental study of optimal thermoelectric load resistance**

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#### **Résumé**

Afin d'arriver à une solution concrète au problème de déchets thermiques industriels, une étude sur la puissance électrique générée et le rendement de conversion de générateurs thermoélectriques a été réalisée. En bref, des modules thermoélectriques disponibles sur le marché furent insérés dans des générateurs thermiquement en parallèle et électriquement en série. L'objectif général de la démarche consiste à obtenir une corrélation permettant de déterminer la résistance électrique externe optimale d'un générateur thermoélectrique afin de correctement les dimensionner en fonction des applications de récupération visées. Cette première étude consistait quant à elle à déterminer si cette résistance électrique externe optimale varie avec les paramètres d'opération.

#### **Abstract**

In an effort to address the problem of industrial thermal waste-heat recovery, the electric power generation capabilities of thermoelectric generators are studied experimentally. To this end, commercially available thermoelectric modules are connected electrically in series and thermally in parallel within the generators and a test stand is built allowing the load resistance to vary during the execution of experiments. The principal objective of this work is to examine the influence of load resistance on the maximum power output of a thermoelectric generator in order to, in further studies, establish a working correlation between the optimal electrical load resistance and the internal electrical resistance of the generator. This work investigates the operating conditions influencing the optimal electrical load resistance when converting waste-heat into electricity by way of a thermoelectric generator.

**Mots-clés :** Générateur thermoélectrique, récupération de déchets thermiques, conversion thermique, résistance externe, module thermoélectrique.

**Key-words:** Thermoelectric generator, waste-heat recovery, thermal conversion, load resistance, thermoelectric module.

#### **1. Introduction**

In part due to recent advancements in thermoelectric materials, in part due to an increased interest in all that is alternative power generation, thermoelectric modules are now being considered in a variety of power generation applications. The purpose of this study is to establish a reliable test facility when optimising the electrical load resistance for maximum thermoelectric power generation. The potential use

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of thermoelectric modules for industrial waste-heat recovery is the principal short term application that is envisioned. There has been significant progress in the field of thermoelectric waste-heat recovery in recent years. In particular, Crane & Jackson (2004) numerically modeled industrial thermal transport in an effort to show that thermoelectric generators, such as the one conceptualized for this study, could work jointly with the heat exchangers of an industrial cooling system. In a later study Sandoz-Rosando & Stevens (2009) detailed the recent and ongoing progresses that have been made in the field of thermoelectric materials showing that systems employing thermoelectric materials are to see significant improvements in the coming years.

Typical industrial waste-heat recovery applications such as pulp and paper mill by-products, data centre cooling systems, nuclear plant cooling systems, etc..., have a residual heat that would remain within a temperature range of 273 K and 323 K. Crane & Bell (2006) and Kajikawa & Onishi (2008), among others, have identified the commercially available thermoelectric semi-conductor Bismuth Telluride,  $\text{Bi}_2\text{Te}_3$  as the most effective material for exploiting the Seebeck effect in this temperature range of application. Recently, Niu *et al.* (2009) optimised a thermoelectric generator designed with electrically connected thermoelectric modules of  $\text{Bi}_2\text{Te}_3$  in an effort to maximize electric power output from low temperature waste fluid. Their study along with those of Rodriguez *et al.* (2009) and Kiran-Ciliz (2003), who measured the efficiency of thermoelectric modules on an individual basis, have shown that it is a timely and a worthy effort, economically and ecologically, to address the possibility of waste-heat to electricity conversion by way of the thermoelectric semi-conductor Bismuth Telluride.

To this end, three liquid to liquid thermoelectric generators and an experimental set up were conceived allowing a variation of all notable parameters in thermoelectric power generation. These were most notably the effect of turbulence due to flow interference, the quantity of  $\text{Bi}_2\text{Te}_3$  used, the temperature difference between hot and cold fluids, the fluid flow rates and – the focus of this study – the effect of load resistance on power output. In particular, the importance in identifying the optimal electrical load resistance (OERL) imposed on a system during thermoelectric power generation is highlighted through a series of tests in which the working parameters are varied.

## 2. Mathematical Model

The objective of the mathematical model presented here, detailed in Hodes (2004), is to illustrate the one-to-one functional dependence between the optimal electrical load resistance (OELR) imposed on the thermoelectric generator and the internal electrical resistance of the generator itself. Consider the case of a temperature gradient due to an influx of heat and a diffusion of heat separated by a distance  $L$  applied to a thermocouple as is illustrated in Fig. 1 below.

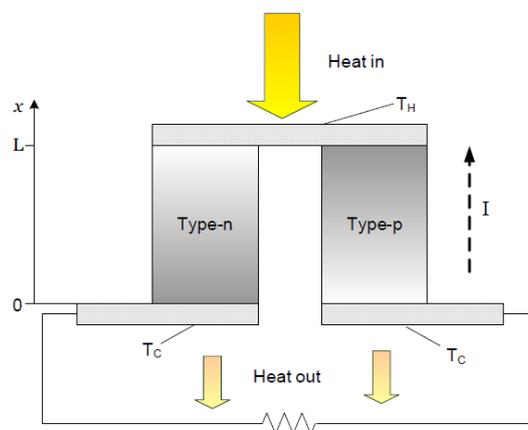


Figure 1. Thermoelectric production from one thermocouple

It is carefully noted that the actual correlation predicting the OELR as a function of internal resistance and the properties of the working materials and fluids is reserved for later studies stemming from this

investigation. The model is one dimensional and approximates the Seebeck coefficient of interconnecting materials (in particular the ceramics holding the thermocouples of the thermoelectric modules together) to be zero (Sandoz-Rosado & Stevens, 2009).

Consider the transformation of heat into electricity by way of the Seebeck effect to be the energy generation component of the heat equation. Applying Ohm's law, the diffusion equation when applied to a single thermocouple, illustrated in Fig. 1, amongst the  $N$  thermocouples of a generator is reduced and integrated with the boundary conditions  $T(0)=T_C$  and  $T(L)=T_H$  providing the following temperature profile (Hodes, 2004).

$$T = -\frac{I^2 \rho}{2kA^2} x^2 + \left( \frac{I^2 \rho L}{2kA^2} + \frac{T_H - T_C}{L} \right) x + T_C \quad (1)$$

In the above,  $T$  is the temperature experienced at a position  $x$ ,  $I$  is the electric current generated,  $\rho$  is the electric resistivity,  $k$  is the thermal conductivity,  $A$  is the surface area of the semiconductor material against the ceramic, and the indices  $H$  and  $C$  indicate the hot and cold sides, respectively. Balancing the heat flux in with the heat flux out and applying Fourier's law of heat conduction, the power output of a thermoelectric module of  $N$  thermocouples is found to be the following.

$$P = -I^2 \left( \frac{2N\rho L}{A} \right) + NI(\alpha_p - \alpha_n)(T_H - T_C) \quad (2)$$

In the above,  $P$  represents the electric power generation, and  $\alpha$  represents the material Seebeck coefficient with the indices  $n$  and  $p$  indicating the n-type and p-type semi-conductors of the thermocouple, respectively. In order to express the power output in terms of the load resistance, it is noted that the term  $2N\rho L / A$  is simply the internal resistance of the set of thermocouples connected in series. Equation 2 is then used to express the load resistance in terms of the internal resistance and then again to express the power output in terms of the load resistance  $R_{load}$  and the internal resistance  $R_{int}$ .

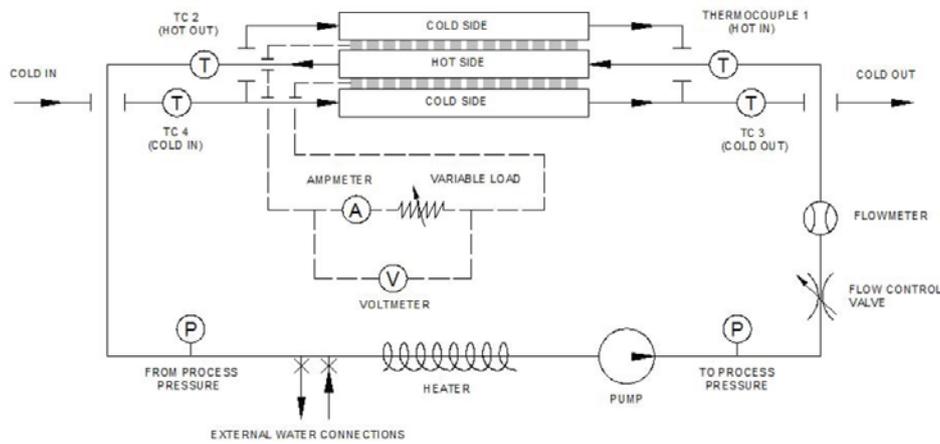
$$P = \frac{(N\Delta T \alpha_{p,n})^2}{R_{int} + R_{load}} - \frac{(N\Delta T \alpha_{p,n})^2 R_{int}}{(R_{int} + R_{load})^2} \quad (3)$$

An analysis of the above equation easily shows that the maximum power output occurs when the electrical load resistance imposed on the system is equal to the internal electrical resistance of the generator. That is,  $R_{load} = R_{int}$ . Admittedly, this is a rather crude estimate but it is important in the sense that it highlights the dependence that the proper choice of load resistance when maximizing thermoelectric production has on the internal resistance of the generator. The experiments summarized below, investigate the OELR's dependence on all working parameters and effectively show a one-to-one functional dependence as is predicted by Eq. 3.

### 3. Experimental Setup

To accommodate three different thermoelectric generators built for this study, the experimental set up illustrated below in Fig. 2 is adopted. It is important to note that the hot and cold inlets are in opposing directions throughout the thermoelectric production. This is noteworthy since often in an attempt to increase the number of thermoelectric modules used in a generator (Niu et al. 2009), the modules are stacked several times requiring alternating parallel and opposing flows. In this study, such a configuration is intentionally avoided for the following reason. In order to fully exploit the Seebeck effect, which is the driving mechanism of the thermoelectric process, a temperature difference must be maintained across the modules containing the semi-conductor  $\text{Bi}_2\text{Te}_3$ . Recalling that for a system that is electrically connected in series, the resulting current will be that of its minimum value along the chain of events. A

thermoelectric generator is therefore limited by the smallest temperature differential it is experiencing. For this reason, any parallel flow, which is favourable to the hot and cold fluids converging in temperature, will decrease the effectiveness of the generator and should be avoided.



*Figure 2. Schematic representation of experimental setup*

In Fig. 2 the thermoelectric modules are illustrated as being pinched between a hot plate injecting heat towards the modules and cold plates diffusing heat away from the modules. The modules themselves consist of thermocouples that are connected electrically in series and thermally in parallel. The thermocouples are simply positively doped and negatively doped  $\text{Bi}_2\text{Te}_3$  semi-conductor pellets physically placed in alternating series. This configuration coupled with the imposed temperature gradient incites an electrical current. This process is known as the Seebeck effect and is in essence the reverse operation of the Peltier effect (Rowe, 2006). What is particularly inviting, from an industrial application perspective, is that the process of harnessing electricity by way of the Seebeck effect described here above does not require any moving parts making the resulting generator virtually maintenance free.

Three thermoelectric generators were designed and built for this study in order to study the working parameters of thermoelectric production. Apart from the inserted thermoelectric modules which consist of Bismuth Telluride pinched between ceramic plates, the generators are entirely made of aluminum. This choice of material was made principally due to the fact that aluminum is an industry standard thereby making this investigation of thermoelectric production relevant to industrial applications. The first of these generators consists of 8 commercially available  $\text{Bi}_2\text{Te}_3$  modules, the second generator consists of 12 modules and the third consist of 40 modules. The largest of the three generators measures 47.22 cm by 9.07 cm by 1.73 cm. The hot and cold fluid flow tubes have an interior diameter of 9.52 mm and an exterior diameter of 12.7 mm.

Typically, such as in Rodriguez et al. (2009) and Niu et al. (2009), when thermoelectric module performance is investigated, a series of experiments is run with the intention of varying operating parameters other than the load resistance (fluid temperatures, flow rate, pressure drop etc...). The effect of load resistance on power output is therefore investigated by studying an ensemble of experiments with differing load resistance. For this reason, there is limited data in the available literature on thermoelectric performance for an evolving load resistance. In order to better understand the effect of electrical load resistance on a thermoelectric generator's ability to produce electricity, it is necessary to perform experiments in which the load resistance is allowed to vary during the execution of experiments. To this end, a rheostat is adapted to the test stand. This apparatus varies the load resistance imposed on the system by compressing or decompressing stacked horizontal carbon plates which are in contact with one another. Compressing the plates together improves the electrical contact between them thereby decreasing the load resistance applied to the system. In this way, the load resistance is allowed to vary from 0 to 100 Ohms with an excellent precession from 0 to 10 Ohms. Data acquisitioning is performed with DataStudio in which the sensors of the test stand are able to communicate directly with the computer via the interface

ScienceWorkshop 750. Styrofoam was used to insulate the generators in order to prevent unwanted thermal diffusion and the entire unit was protected with Plexiglas for protection from possible hot fluid ruptures.

Finally, the influence of turbulence on the OELR is investigated by examining the effects of flow impeding inserts. Three different inserts, illustrated in Figure 3, were built in order to be inserted into the fluid carrying flow tubes. The first of these inserts is spiral, where as the second and third have alternating tabs that are  $5/8^{\text{th}}$  of an inch and  $5/16^{\text{th}}$  of an inch apart, respectively.



Figure 3. Flow impeding inserts

### 3. Results and Discussion

Experiments were run for the three generators described above recalling that their principal differing characteristic is the number of thermoelectric modules within each one. The power output of each generator was recorded in a series of tests. The tests differed by changing the temperature gradient, the presence of flow impeding inserts, and the fluid flow rates. During each one of these tests, the electrical load resistance was varied thereby recording the power output evolution with respect to a changing load resistance. In this way, the OELR is identified as being the load resistance providing the maximum power output. A small sample of the results is provided in Fig. 4 in which the power performance of two of the generators is illustrated as a function of load resistance. In these two sample experiments, the hot flow rate is 5 L/min, the cold flow rate is 10 L/min and the flow impeding inserts with tabs at every  $5/16^{\text{th}}$  of an inch are present in all flow tubes. It is important to note that the Reynold's number with characteristic length being the diameter of the flow tubes  $Re_D = 4\dot{V}\rho_l / \pi D\mu$  for the set of experiments performed in this study ranged from 11598 and 112 285 placing all experimental Reynold's numbers well above 2300. Therefore, all flow experienced in all tests, regardless of any presence of flow impeding inserts, are well within the turbulent regime.

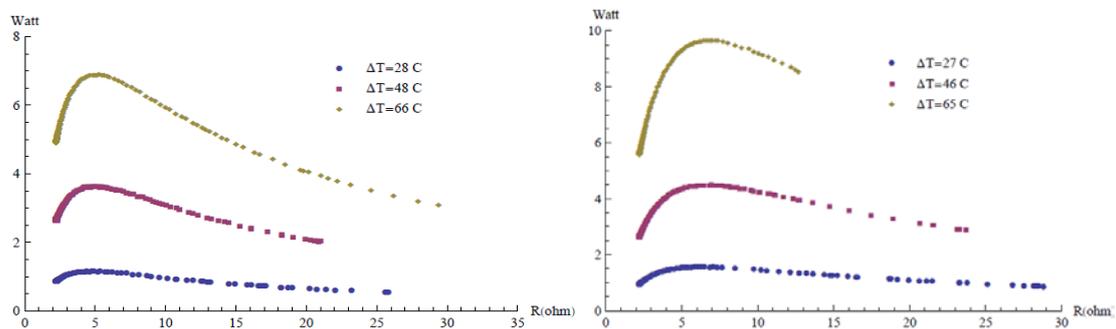


Figure 4. Sample of power output results with respect to load resistance (left) from a generator containing 8 thermoelectric modules and (right) a generator containing 12 thermoelectric modules

These above results show that the OELR providing a maximum power output is invariant with respect to the temperature gradient. This is significant since it is this temperature gradient that is inciting the Seebeck effect. An optimal load resistance can therefore be easily identified in more manageable low temperature conditions and then applied in higher temperature applications. It is duly noted that a poor choice in load resistance has a very negative effect on power output as is illustrated in Fig.4 by the sharp

increase in power as the optimal load resistance is approached. Similar results are found for other experiments (not presented herein) in which the optimal load resistance is identified for flow with or without flow impeding inserts. Despite that the flow impeding insert with tabs at every 5/16<sup>th</sup> of an inch greatly improved the power output; the choice of optimal load resistance remained unchanged for all inserts tested and in the absence of any inserts. Furthermore, other results (not shown here) show that the flow rates, and therefore the heat injection and heat diffusion rates, had no influence on the OELR. Finally, the only influencing parameter on the OELR observed through experimentation was the number of thermoelectric modules contained in the generator.

### 3. Conclusion

In this investigation, thermoelectric generators are designed, built, and tested in an effort to better understand the influence of the working parameters on thermoelectric power generation. In particular, an experimental setup is constructed that allows the electrical load resistance to vary during each experiment. Analysis of the experimental results shows that the most optimal electrical load resistance maximizing thermoelectric power output is invariant under all operating conditions and only varies when the number of thermoelectric modules within the generator varies. In conclusion, the choice of load resistance that should be applied when maximizing power output is solely dependent on the internal resistance of the generator. This implies that for any one generator, favourable operating conditions may be used when identifying the optimal load resistance which is then subsequently the load resistance to be used for that generator under any operating conditions. Finally, by experimentally validating the OELR one-to-one functional relationship with a generator's internal resistance, this study provides the stepping stone necessary for future works to develop a working correlation identifying the OELR for any one generator as a function of the generator's internal electrical resistance.

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### Acknowledgment

The first author gratefully acknowledges the support from the Ministère d'éducation, du loisir et du sport of Québec and the NATEQ through grant number PART2009N003. The second author thanks the NSERC for a Discovery grant. We gratefully thank Stéphane Piché, Rémi Pelletier, Éric Sempels and Nathaniel Lalande-Bertrand for their professionalism and diligence throughout this study.

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