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Mixed convection in a vertical porous channel: heat transfer enhancement

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Abstract

Mixed convection heat transfer in a vertical porous channel heated by the wall and isolated on the other face was simulated numerically. The porous medium is formed by a solid matrix of spherical beads. The fluid considered is air that saturates the solid matrix. The two-temperature model and the Darcy-Brinkman-Forchheimer equation are adopted to represent this system and the porosity is considered as variable within the domain. The numerical model was used to analyze the effect of several operating parameters on heat transfer enhancement. Heat transfer decreases with the increase of the form factor. When Biot number increases, heat transfer between the heated wall and the porous domain correspondingly increases. Heat transfer increases with Reynolds number and with the thermal conductivity of the solid matrix. The influence of the heat conductivity of the particles on the heat transfer of air in the porous medium decreases when the conductivity of the particles increases, and that especially when the diameter of the beads is large. Finally, the Nusselt numbers based on the particle diameter have been correlated with respect to the Reynolds number for glass spheres and for metallic particles.

Keywords: vertical channel, porous medium, heat transfer, mixed convection .

Nomenclature

A	geometrical form factor
a_{fs}	solid-fluid exchange area, m^{-1}
c_p	specific heat at constant pressure, Jkg^{-1}K^{-1}
d	diameter of the beads, m
g	gravity acceleration
H	height of the channe, m
h_{fs}	fluid-solid convective transfer coefficient
K	permeability,m^2
L	width of the channel, m
P	pressure, Pa
q_w	heat flux density at the wall, Wm^{-2}
T	temperature, K
u,v	velocity components, m,s^{-1}

Greek Symbols

ε	porosity
ρ	mass density, kgm^{-3}
θ	dimensionless temperature
Λ	inertia coefficient
β	thermal expansion coefficient, K^{-1}
λ	heat conductivity Wm^{-1}K^{-1}
μ	dynamic viscosity Pa.s

Subscripts

0	reference value
f	fluid (air)
s	solid
m	mean
w	wall
p	particle

Notations

Non-dimensional Numbers

Re_p	Reynolds number, [$\rho UD/\mu$]
Bi	Biot number related to the fluid-solid exchanges
Da	Darcy number
F	Forcheimer coefficient
Gr	Grashoff number
Nu	Nusselt number for the fluid
Pr	Prandlt number
r_{dL}	ratio between the diameter of the particles and the width of the channel

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1. Introduction

Flows and heat transfer in porous media have been largely studied numerically and experimentally due to the importance of their applications such as in energy extraction and storage, heat exchangers, petrochemical processes, thermal insulating of buildings, geophysical applications, etc. The physical and geometrical characteristics of a porous medium constituted by a packing of spherical beads are important parameters for the study of heat transfers in such media. Vafai and Tien [1] studied the flow and the heat transfer in a porous medium limited by a solid horizontal wall. They showed that the wall and inertia effects are more noticeable in highly permeable media, for large Prandtl numbers, for large pressure gradients and in the region close to the leading edge of the flow boundary layer. Lee et al. [2] studied numerically the influence of the thickness of the dynamic limit layer on the heat transfer in a channel filled with a porous medium. They obtained a correlation relating that thickness to the Darcy and Reynolds numbers based on the permeability of the medium. They showed that the decrease of that thickness increases the Nusselt number. Jiang et al. [3] performed an experimental study on a horizontal channel in forced convection with water as the coolant, which showed that the heat transfer coefficient increases with the diameter of the glass particles and decreases with the diameter of metal particles. Hwang and Chao [4] studied experimentally and numerically the heat transfer of air in a porous medium and showed that the model with one-energy equation overestimates the Nusselt number. In the case of glass beads filling a horizontal channel, Jiang et al. [5] showed that the heat transfer increases when the diameter of the particles decreases and when the conductivity of the particles increases. The influence of the thermal conductivity of solid particles on the convection heat transfer of air in porous media decreases with increasing solid particle thermal conductivity.

This literature review shows that most of the works performed on porous media concern domains limited by horizontal walls and consider liquids as coolants. In addition, the influences of the form factor and of the fluid-particles thermal exchanges on the heat transfer are not considered in those studies. The present work concerns mixed convection in a vertical channel totally filled by a porous medium, crossed by air as the heat transfer fluid and heated on one of the walls by a constant heat flux density. According to our knowledge, this has never been studied before.

The main objective of the present study is to analyze the effect of several operating parameters on heat transfer enhancement in porous media. The influences of the variation of Reynolds number, Biot number, the form factor, the diameter of the beads, and the thermal conductivity of the solid matrix on heat transfer, need to be further studied to optimize heat transfer enhancement using porous media.

2. Content

The study concerns the flow and heat transfer in a vertical porous medium constituted by two phases: a fluid phase of air and a solid phase of spherical beads (*Figure 1*).

The considered porous domain is a channel open at both extremities, filled with homogeneous spherical beads, saturated by air as the fluid. The set of equations that rules the flow and the transfers in the studied domain is formed by the continuity equation, the energy equations for the fluid and solid phases and the momentum based on the model of Darcy-Brinkman-Forcheimer with inertia and viscosity

$$\nu = \frac{\mu}{\rho}\nabla\cdot\langle V\rangle = 0 \qquad (1)$$

$$\frac{\rho}{\varepsilon}\frac{\partial\langle V\rangle}{\partial t} + \frac{\rho_f}{\varepsilon}\left\langle V\cdot\nabla\frac{V}{\varepsilon}\right\rangle = \nabla\langle p\rangle^f + \rho g - \frac{\mu_f}{K}\langle V\rangle - \frac{\rho_f F\varepsilon}{\sqrt{K}}\left[\langle V\rangle\langle V\rangle\right]J + \frac{\mu_f}{\varepsilon}\nabla^2\langle V\rangle \qquad (2)$$

$$(1-\varepsilon)\rho_s C_{ps}\frac{\partial T_s}{\partial t} = \nabla\cdot\left(\lambda_{s,eff}\nabla\langle T_s\rangle^2\right) + h_{fs}a_{fs}\left(\langle T_f\rangle^f - \langle T_s\rangle^s\right) \qquad (3)$$

$$\varepsilon\rho_f C_{pf}\frac{\partial T_f}{\partial t} + \rho_f C_{pf}\langle V\rangle\cdot\nabla\langle T_f\rangle^f = \nabla\cdot\left(\lambda_{f,eff}\nabla\langle T_f\rangle^f\right) - h_{fs}a_{fs}\left(\langle T_f\rangle^f - \langle T_s\rangle^s\right) \qquad (4)$$

The characteristic parameters used in equations (1)-(4) are:

$K = \dfrac{d^2\varepsilon^3}{150(1-\varepsilon)^2}$ permeability of the porous medium,

$F = \dfrac{1.75}{150\varepsilon^{1.5}}$ Forchheimer coefficient,

$h_{fs} = \dfrac{\lambda_f}{d}\left[2 + 1.1\Pr_f^{0.33}\left(\dfrac{vd}{v_f}\right)^{0.6}\right]$ fluid-solid convective heat transfer coefficient, $a_{fs} = \dfrac{6(1-\varepsilon)}{d}$ fluid solid contact surface.

The porosity in the channel is from [6]

$$\varepsilon = \varepsilon_\infty\left[1 + 1.7\exp\left(\frac{-6x}{d}\right)\right] \qquad \text{if } 0 \le x \le \frac{L}{2} \text{ and}$$

$$\varepsilon = \varepsilon_\infty\left\{1 + 1.7\exp\left[\frac{-6(L-x)}{d}\right]\right\} \qquad \text{if } \frac{L}{2} \le x \le L$$

The boundary conditions are the following:

$$y = 0 : u = 0, v = V_0, T_f = T_0, \lambda_{s,eff}\frac{\partial T_s}{\partial y} = h_{sf}(T_s - T_0)$$

$$y = H : \frac{du}{dy} = \frac{dv}{dy} = 0, \frac{\partial T_s}{\partial y} = \frac{\partial T_f}{\partial y} = 0$$

$$x = L : u = v = 0, \frac{\partial T_s}{\partial x} = \frac{\partial T_f}{\partial x} = 0$$

$$x = 0 : u = v = 0, q_w = -\lambda_f\left(\frac{\partial T_f}{\partial x}\right) = -\lambda_s\left(\frac{\partial T_s}{\partial x}\right) = 0$$

T_0 et V_0 are respectively the temperature and the velocity at the entrance of the porous domain.

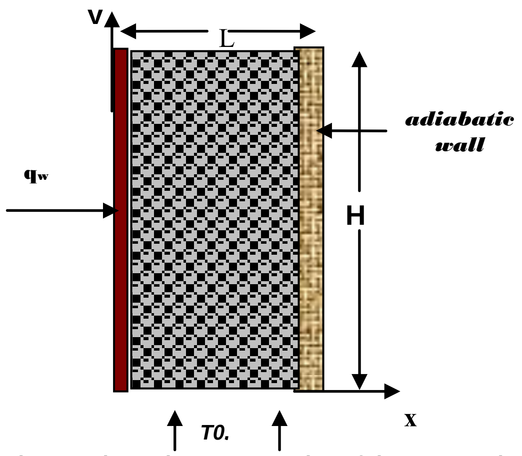

Fig. 1. Schematic representation of the porous channel

The equations under dimensionless form become

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \qquad (5)$$

$$\frac{1}{\varepsilon}\left[\frac{\partial(UU/\varepsilon)}{\partial X}+\frac{\partial(UV/\varepsilon)}{\partial X}\right]= -\frac{\partial P}{\partial X}-\left(\frac{r_{dL}}{\mathrm{Re}_p\,\mathrm{Da}}+\Lambda_L\left|\overrightarrow{W}\right|\right)U +\frac{r_{dL}}{\varepsilon\,\mathrm{Re}_p}\left(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\right) \qquad (6)$$

$$\frac{1}{\varepsilon}\left[\frac{\partial(UV/\varepsilon)}{\partial X}+\frac{\partial(VV/\varepsilon)}{\partial Y}\right]= -\frac{\partial P}{\partial Y}-\left(\frac{r_{dL}}{\mathrm{Re}_p\,\mathrm{Da}}+\Lambda_L\left|\overrightarrow{W}\right|\right)V +\frac{r_{dL}}{\varepsilon\,\mathrm{Re}_p}\left(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right)+\frac{r_{dL}\mathrm{Gr}_f}{\mathrm{Re}_p^{\,2}}\theta_f \qquad (7)$$

$$0=\frac{r_{dL}(1-\varepsilon)}{\mathrm{Pr}_f\,\mathrm{Re}_p}\left[\frac{\partial}{\partial X}\left(\frac{\partial\theta_s}{\partial X}\right)+\frac{\partial}{\partial Y}\left(\frac{\partial\theta_s}{\partial Y}\right)\right] +\frac{\lambda_f}{\lambda_s}\frac{r_{dL}\mathrm{Bi}}{\mathrm{Pr}_f\,\mathrm{Re}_p}\left(\theta_f-\theta_s\right) \qquad (8)$$

$$\left[U\left(\frac{\partial\theta_f}{\partial X}\right)+V\left(\frac{\partial\theta_f}{\partial Y}\right)\right]= \frac{r_{dL}\varepsilon}{\mathrm{Pr}_f\,\mathrm{Re}_p}\left[\frac{\partial}{\partial X}\left(\frac{\partial\theta_f}{\partial X}\right)+\frac{\partial}{\partial y}\left(\frac{\partial\theta_f}{\partial Y}\right)\right]-\frac{r_{dL}\mathrm{Bi}}{\mathrm{Pr}_f\,\mathrm{Re}_p}\left(\theta_f-\theta_s\right) \qquad (9)$$

These equations involve the relevant dimensionless parameters of the problem namely: Gr_f, Pr_f, Da, Λ_L, Re_p and Bi, respectively the Grashoff number, the Prandlt number, the Darcy number, the inertia coefficient, the Reynolds number, and the Biot number related to the fluid-solid exchanges.

The dimensionless numbers defined in equations (5)-(9) are defined by

$$\mathrm{Gr}_f=\frac{g\mathrm{B}\dot{q}_w L^4}{v_f^2\lambda_f};\ \mathrm{Pr}_f=\frac{v_f}{\alpha_f};\ Da=\frac{K}{L^2};\ \Lambda_L=\frac{FL}{K};$$

$$\mathrm{Re}_p=\frac{V_0 d}{v_f};\ \mathrm{Bi}=\frac{h_{fs}a_{fs}L^2}{\lambda_f};\ r_{dL}=\frac{d}{L};\ \lambda=\frac{\lambda_s}{\lambda_f}$$

where the parameters r_{dL} and λ are respectively the ratio of the diameter of the particles to the width of the channel, and the ratio of the solid to the fluid conductivities.

The local Nusselt numbers related to the fluid and solid phases are given by the following expressions

$$Nu_f=-\frac{L}{T_{fw}-T_{fm}}\left(\frac{\partial T_f}{\partial x}\right)_w,\quad Nu_s=-\frac{L}{T_{sw}-T_{sm}}\left(\frac{\partial T_s}{\partial x}\right)_w$$

where the mean temperatures are given by

$$T_{fm}=\frac{1}{U_m}\int_0^L vT_f dx;\ T_{sm}=\frac{1}{L}\int_0^L T_s dx;\ U_m=\frac{1}{L}\int_0^L v dx$$

The mean values of the Nusselt number are

$$\mathrm{Nu}_{fm}=\frac{1}{H}\int_0^H \mathrm{Nu}_f dy;\ \mathrm{Nu}_{sm}=\frac{1}{H}\int_0^H \mathrm{Nu}_s dy;$$

$$\mathrm{Nu}_{tm}=\mathrm{Nu}_{fm}+\mathrm{Nu}_{sm}$$

Several authors use a Nusselt number based on the diameter of particle, which would be expressed in the present case of the total mean Nusselt number as:

$$\mathrm{Nu}_p=\mathrm{Nu}_{tm}r_{dL}.$$

3. Results and discussion

The set of nonlinear partial differential equations is solved numerically by the finite volume method [7]. A non-uniform grid in both directions is used. The coupling velocity-pressure is solved by the algorithm SIMPLE, and a power-law scheme is adopted to express the convective terms. The convergence criterion is satisfied when the temperature or velocity field difference between two successive iterations is lower than 10^{-6}.

3.1. Influence of Biot Number

In this study, Biot number characterizes heat exchanges between the fluid phase and the spherical particles The influence of Biot number on the mean total Nusselt number is represented in two cases, the first one where the inlet velocity in the channel varies and the bead diameter is fixed (r_{dL}=0.03 m, Fig.2), the second one where the bead diameter varies and the inlet velocity in the channel is fixed (V_0=0.4 m/s, Fig.3). When heat exchange between air and the beads increases, causing an increase of Biot number, the total Nusselt number increases (linearly if the inlet velocity is varied and the bead diameter is fixed), i.e. heat transfer between the heated wall and the porous domain is increased. The increase of Biot number is mainly caused by an increase of the inlet velocity or a decrease of the bead diameter, which induces an increase of heat transfer between the heated wall and the porous medium, as previously discussed. Consequently, an intensification of heat transfer between the fluid and the particles provokes an increase of transfer between the heated wall and the porous medium.

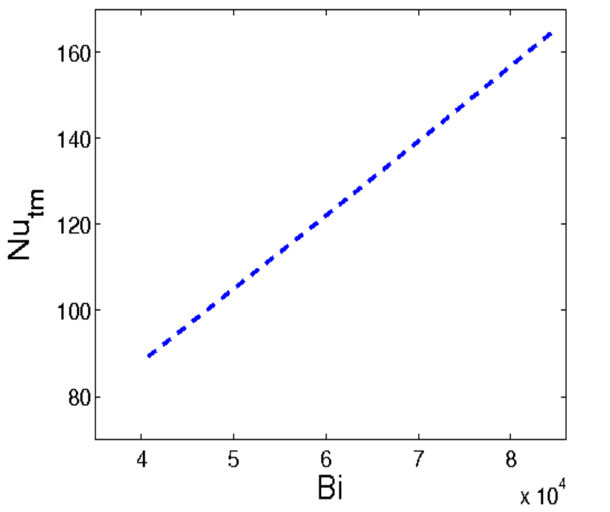

Fig. 2. Variation of the total Nusselt number with respect to Biot number at constant bead diameter ($Gr_f=10^7$, $r_{dL}=0.067$, λ =28.57, A=5)

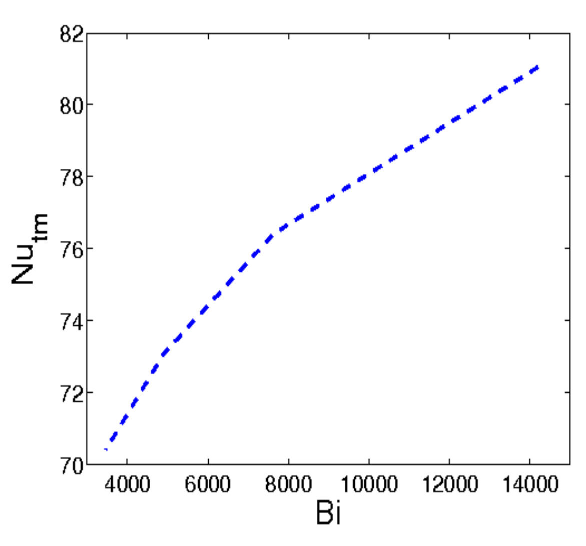

Fig3. Variation of the total Nusselt number with respect to Biot number at constant inlet velocity ($Gr_f=10^7$, $r_{dL}=0.067$, λ =28.57, v=0.04m/s, A=5)

3.2 Influence of the form factor

The heat transfer increases when the form factor decreases (Figure 4). A relative increase of the channel height generates a heating of the medium, as the exchange surface between the fluid and the heated wall increases, and consequently the temperature difference between the heated wall and the porous medium decreases which induces a decrease of the heat transfer.

3.3 Influence of the Reynolds number

The local heat transfer decreases along the heated wall in the flow direction whereas it increases with the Reynolds number (Figure 5).

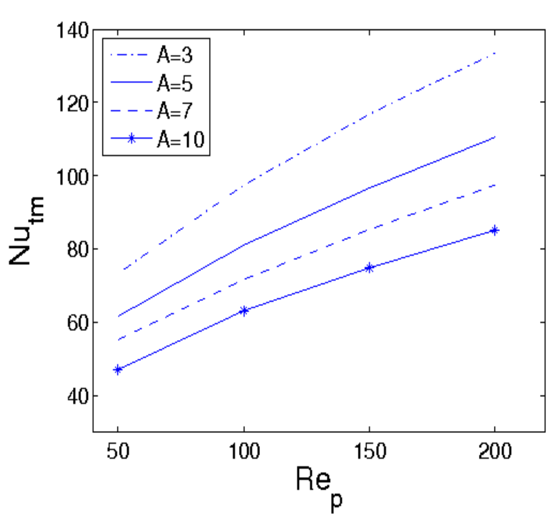

Fig 4. Variation of the mean fluid Nusselt number with respect to the Reynolds number for different form factors ($Gr_f=10^7$, $r_{dL}=0.067$, λ =28.57)

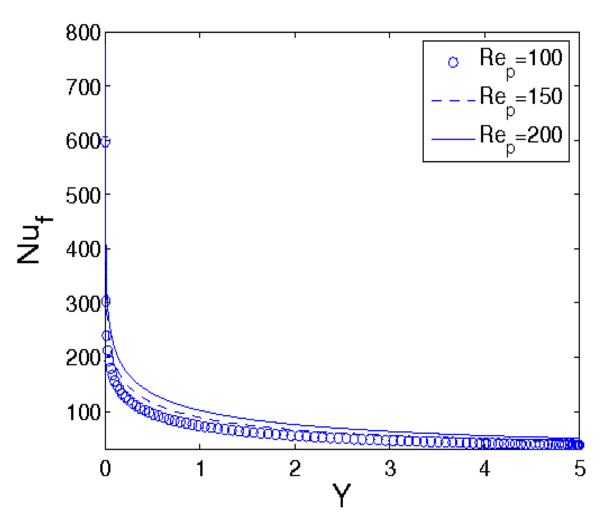

Fig 5. Variation of the local fluid Nusselt number at X=0, for different Reynolds numbers

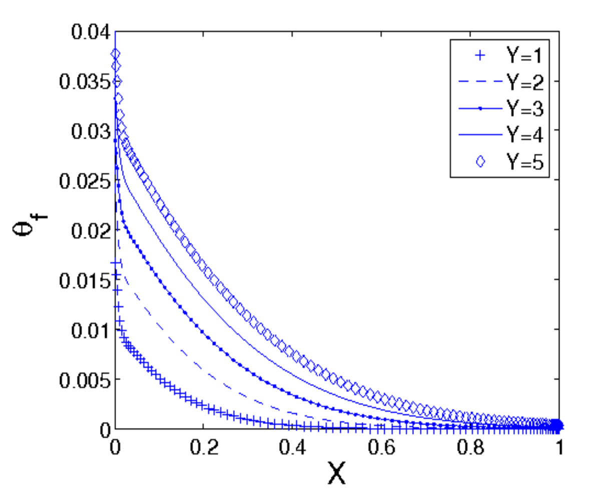

Fig 6. Variation of the temperature of the fluid phase, at different heights in the porous channel

This occurs, as when the flow velocity increases, the thickness of the limit layer formed along the heated wall decreases and consequently the heat exchanges between the wall and the fluid increase. An important variation of the local Nusselt number close to the entrance of the channel can also be noticed while it remains nearly constant in the remaining domain which is in agreement with [3]. In the neighborhood of the channel entrance, the temperature deviation between the fluid and the wall is relatively large, which results in large values of the Nusselt number (Figure 6).

3.4 Correlations proposed

The heat transfer coefficient between the wall and the air increases with the air velocity (figure 7).

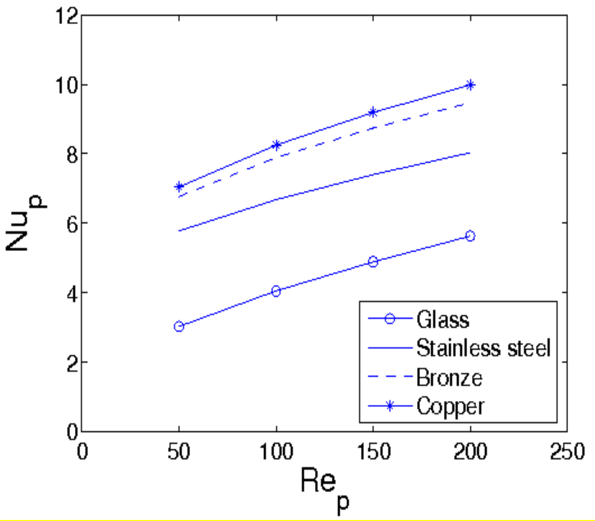

Fig 7. Variation of the fluid mean Nusselt number with respect to the Reynolds number for different conductivities of beads ($Gr_f=10^7$, A=5, r_{dL}=0.034)

Correlations of Nusselt number with respect to Reynolds number r_{dL}=0.034 and for 50≤ Re_p≤200 are proposed,

- for the glass particles $Nu_p = 0.52\,Re_p^{\,0.44}$
- for the metal particles $Nu_p = a\,Re_p^{\,b}$

with a= 2.28 for stainless steel, 2.62 for bronze and 2.63 for copper and b= 0.23 for stainless steel, 0.24 for bronze and 0.25 for cooper.

A nonlinear correlation equation for the glass particle, in term of Re_p and L/d is obtained as $Nu_p = a\,Re_p^{\,b}(L/d)^c$ for 50≤ Re_p≤200:

$$Nu_p = 2.5\,Re_p^{\,0.41}\left(\frac{L}{d}\right)^{-0.41}$$

4. Conclusion

A numerical study dealing with the heat transfer in a vertical porous channel with a variable porosity has been conducted for a two-temperature model. This work shows that the heat transfer increases with Reynolds number, decreases when the form factor increases, increases with the decrease of the diameter of the metallic or glass beads. The heat transfer in the case of a matrix formed by metallic beads is larger than by glass beads. This influence of the heat conductivity of the particles on the heat transfer of air in the porous medium decreases when the conductivity of the particles increases especially when the diameter of the beads is large. The Nusselt numbers based on the particle diameter have been correlated with respect to the Reynolds number for glass spheres and for metallic particles.

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