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Wavelet-based parameterization of control variables in diffuse optical tomography

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Abstract

The present paper deals with the use of wavelet theory in the ill-posed inverse problem of diffuse optical tomography. The rationale behind this choice is to exploit the filtering potential of wavelets to the noisy cost function gradient, the latter being an essential ingredient in the optical properties reconstruction process. The proposed algorithm is based on the Broyden-Fletcher-Goldfarb-Shanno (BFGS) method with an inexact line-search method, where the continuous cost function gradient is computed with the adjoint state method. After decomposition of this gradient with the discrete wavelet transform, filtering is performed through a thresholding rule on the detail coefficient vectors.

Key words: BFGS, filtering, inverse problem, parameterization, wavelet
MSC 2010: 78M50, 65T60, 35Q93

1 Introduction

The Optical Tomography (OT) consists in reconstructing optical property maps of heterogeneous semi-transparent media from radiative measurements obtained with the help of sources and sensors located on the edges of the medium. Two mathematical descriptions are commonly used to predict the propagation of optical radiation through the participating medium, namely the Radiative Transfer Equation (RTE) based model and the Diffuse Approximation (DA) model. The DA model, which is considered in this paper, assumes that the media is poorly absorbing and highly diffusing. Applications of the OT with the DA model typically concern the detection of cancerous tumors in tissues such as the breast. It is also planned to use such method to characterize radiative properties of materials such as metallic open-cell foam, allowing the systematic design of materials in thermal and energetics engineering.

The forward model associated to the diffuse optical tomography problem is given by [1]:

$$-\nabla \cdot \left([n(\kappa+\sigma)]^{-1} \nabla \varphi \right) + \left(\kappa + \frac{2i\pi\nu}{c} \right) \varphi = 0 \quad \text{in } \mathcal{D} \tag{1}$$

$$\varphi + \frac{A}{2\gamma} [n(\kappa+\sigma)]^{-1} \nabla\varphi \cdot \boldsymbol{n} = \frac{I}{\gamma} \mathbb{1}_{[\zeta \in \partial\mathcal{D}_s]} \quad \text{on } \partial\mathcal{D} \tag{2}$$

with $\varphi : \mathcal{D} \to \mathbb{C}$ is the photon density, κ and σ are, respectively, the absorption and reduced scattering coefficients, and I denotes the prescribed radiative intensity on $\partial\mathcal{D}_s$. Parameters γ, which only depends on the dimension of $\mathcal{D}$, n, and A, which characterizes the reflection at the boundary, are given. Finally, ν is the source modulation frequency and c is the velocity of the light. The difference, in the least squares sense, on sensor locations between the state φ and experimental measurements is integrated to a cost function j to be minimized. Thus, the solution of the inverse problem reads: "*Find the functions κ^* and σ^* such that $j(\kappa^*, \sigma^*) < v$*", where v integrates variances of errors for all measurements.

2 Optimization

The minimization of the cost function is carried out by the Broyden-Fletcher-Goldfarb-Shanno algorithm associated to a fast inexact line-search. This algorithm relies on the gradient computed through the solution of the additional adjoint problem. In [2], the cost function gradient are written in a continuous way before being discretized choosing a finite element basis for the parameterization of the optical properties. These continuous gradients are given by:

$$\nabla^{\kappa} j(\boldsymbol{x}) = \Re\left(\varphi(\boldsymbol{x})\varphi^*(\boldsymbol{x}) - n\left[n(\kappa+\sigma)\right]^{-2} \nabla\varphi(\boldsymbol{x}) \cdot \nabla\varphi^*(\boldsymbol{x})\right) \quad (3)$$

$$\nabla^{\sigma} j(\boldsymbol{x}) = -n\left[n(\kappa+\sigma)\right]^{-2} \Re\left(\nabla\varphi(\boldsymbol{x}) \cdot \nabla\varphi^*(\boldsymbol{x})\right) \quad (4)$$

3 Filtering

In reality, the computed gradients present high frequency fluctuations due to the noise coming from measurements and propagating through the adjoint variable. So far, regularization strategies concerned: (i) parameterization based on a coarser mesh, performed using projection of the state and adjoint variables on the coarse mesh, and (ii) the use of Sobolev inner products when extracting the cost function gradient. The strategy implemented in this paper is based on the wavelet decomposition of functions $f \in L^2(\mathbb{R}^n)$ applied to the cost function gradients. Concretely, the gradient maps are discretized with 2^{nJ} values. Then, the dyadic wavelet transform [3] is computed for scales 2^j, $j = 1, \ldots, J$. A thresholding rule is finally considered on the detail coefficient vectors before reconstructing the filtered gradient. The study concerning the selection of the wavelet and thresholding method is in progress.

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