14

Cost Analysis of Space Exploration for an Extraterrestrial Civilization

Yvan Dutil and Stéphane Dumas

1.0. Introduction

Are some civilizations bound to their home planet because the local gravitational field is too strong? Can space exploration be so difficult that a civilization might just give up before even trying? Are we favored compared to our galactic neighbors? Analysis of the relative cost of space exploration between planets, in light of the physical characteristics of a civilization's planet of origin, might provide some insight into the problem of the Fermi paradox or in a more general way the possibility of physical contact between civilizations. This is not a trivial issue, since there may be some advantage to using space probes as an effective way to communicate between civilizations (Rose and Wright 2004). Such contacts do not need to be done by sentient beings themselves, as advanced automated space probes could achieve the same goal (Bracewell 1960; von Neumann and Burks 1966; Boyce 1979).

Nevertheless, even classical SETI can benefit from space exploration capabilities. For example, it would be helpful to place a SETI observatory on the lunar far side to avoid local radio interference (Heidmann 1994). Ironically, spacefaring capabilities would make this type of SETI project even more attractive, as interference shielding is more complex when interfering sources are themselves in space. Alternatively, an extraterrestrial civilization might want to take advantage of the gravitational focusing of the Sun to increase the sensitivity of its SETI project, which would require significant spacefaring capabilities, as noted in the immediately preceding chapter in this volume (Maccone 2011).

We are constructing our analysis around a simple and sound scenario. Before exploring interstellar space, a civilization must first succeed in achieving two earlier steps of exploration: getting above the atmosphere in the lowest possible orbit and moving between planets within their own stellar system. Once these steps have been mastered, there will be little variation in interstellar mission costs. In consequence, if the cost of launching a satellite in low orbit and reaching another nearby planet is prohibitive, an extraterrestrial civilization might simply give up space exploration and direct its resources elsewhere.

To shed some light on this potential problem, we analyze the relative energy cost of launching a satellite in low Earth orbit and from there reaching the nearest planet, in various potential life-bearing planetary configurations.

2.0. Budgeting Space Exploration

From the engineering point of view, the cost of space exploration is essentially determined by one parameter: the velocity change. Velocity changes themselves are determined by the local gravity field and the maneuvers needed to move from one orbit to another. Three basic maneuvers are needed to perform interplanetary exploration: (1) getting above the atmosphere, (2) circularizing the orbit, and (3) reaching another planet's orbit. It should be noted that leaving the home planet's gravity field requires only $\sqrt{2} = 1.4142$ times the velocity change needed to reach low earth orbit. The same ratio also applies to the problem of escaping the stellar gravity field. In consequence, only those three basic maneuvers need to be analyzed.

If the velocity change is an important factor, it is because it drives the total weight of the launcher. As more velocity is needed, more and more fuel must be carried in addition to the useful payload. The relation between the velocity change (ΔV) and the mass fraction (original mass over the spacecraft mass m_o/m_s) is known as the rocket equation. This equation was derived for the first time by the Russian space exploration pioneer Konstantin Tsiolkovsky in 1903 (Tsiolkovsky 1903). It has this form:

$$\frac{m_o}{m_s} = \exp\left(\frac{\Delta V}{Ve}\right)$$

where V_e is the ejection speed of rocket exhaust.

Since for a chemical rocket, ejection speed is controlled by characteristics of the reactants involved, this parameter can be assumed to be relatively constant for every civilization. Indeed, one of the most effective practical reactions is between hydrogen and oxygen—two very abundant elements forming the water molecule. Alternate propulsion methods, such as ionic and nuclear motors, have higher ejection speeds but are impractical for first stage launch, since they do not provide enough thrust to fight the local gravity. Once in space they are more useful, but take a long amount of time to be effective. Use of such an advanced space propulsion system will concentrate the expense of cost on the launch to low orbit. To avoid unneeded complexity, we will assume that all space maneuvers use chemical propulsion. As we will see later, this assumption bears little impact on our conclusions.

Another factor driving the cost is the size of the smallest possible spacecraft. Technology drives the minimal size of artificial satellites. It is reasonable to assume that the dry weight of a satellite for a given function is relatively constant for any planet, because aerospace technologies have performances very close to those allowed by physics. However, for inhabited spacecraft, the minimal size is driven by the size of the pilot. Unfortunately, we cannot derive the pilot size from first principles based on planetary and stellar parameters. This is why, and for the stake of simplicity, we will assume that the reference spacecraft mass is independent of the local condition.

Before going into orbit, a spacecraft must first get above the atmosphere. Hence, the minimum orbital altitude is controlled by the atmospheric drag. Atmospheric density and the shape and velocity of the satellite are the main factors affecting the drag. As we will see later, orbital velocity does not change much across planets. We also assume that satellite size and shape are constant. In consequence, only the atmospheric density really matters.

Atmospheric density drops exponentially with the altitude. On Earth, for an increase of 7.6 km, the scale height, the atmospheric density drops by a factor of 2.72 on average. The scale height itself is proportional to the atmospheric temperature and inversely proportional to the local gravitational acceleration and average mass of atmospheric molecules. As we expect from habitability considerations, atmospheric composition and temperature will not be vastly different from the Earth's, and scale height will mostly depend on local gravity.

The cost of climbing above the atmosphere is proportional to the energy needed, which is the product of the height by the surface gravitational acceleration. Since scale height is inversely proportional to the surface acceleration, the product of the two is roughly constant for any inhabited planet. Surface pressure also affects the minimal orbital altitude, but its impact is minimal. For example, going up by one scale height will reduce the atmospheric density by a factor of 2.72. Since the lowest orbital attitude is at least twenty scale heights, a small change in altitude can absorb a wide range of surface atmospheric pressure, 0.3 to 3 times Earth's, with minimal impact on the launching cost. In addition, for typical launch system, fighting gravity is only 20 to 25 percent of the velocity change needed to reach the Earth orbit; the cost of going above the atmosphere is essentially independent of the planet of origin. Impact of atmospheric drag is even lower, as it is roughly equal to 10 percent of the gravity for our launcher technology, and optimization of the launch profile minimizes its impact on mission performances.

Clearing the atmosphere is a relatively easy step compared to reaching orbital velocity. To reach orbit, we must reach a velocity that will generate enough centrifugal acceleration to balance the local gravitational acceleration. The following equation describes the relation between the two phenomena:

$$g = \frac{v^2}{R+h} \approx \frac{v^2}{R} \text{ when } h << R.$$

It is reasonable to assume that the minimal orbital altitude (*h*) is much smaller than the planetary radius (*R*) as it is in the case for Earth. For a given planet, the surface gravitational acceleration (*g*) is proportional to the planet's mass and inversely proportional to the square of its radius: $g \propto \frac{M}{R^2}$.

We can use this formula to estimate the ΔV needed to put a satellite in orbit. Using the previous result, we derive:

$$\Delta V \propto \sqrt{\frac{M}{R}}.$$

At this point in our analysis, we need to establish a relationship between the planet's mass and radius. Assuming a constant density, the planetary radius is then proportional to the cubic root of the volume and, as a consequence, the mass ($R \propto \sqrt[3]{M}$). This approximation is roughly correct between 0.3 and 10 Earth masses (Seager et al. 2007; Swift et al. 2010), which covers the range expected for an inhabited exoplanet. In consequence:

$$\Delta V \propto \sqrt{\frac{M}{R}} \propto \sqrt{\frac{M}{M^{1/3}}} = M^{1/3}$$

The relation indicates that the velocity change needed to reach low planetary orbit is only weakly sensitive to the planet's mass, as velocity change only increases with the cubic root of the planet's mass.

Integrating this result in the cost relation derived previously, we have:

$$\boxed{\text{cost} \propto \exp(\Delta V) \propto \exp\left(M^{1/3}\right)}$$

In consequence, the relative launch cost compared to the Earth for a 0.3 earth mass ($0.3M_{\oplus}$) planet would be 0.72 and 1.56 for a three earth mass ($3M_{\oplus}$) planet. Keep in mind that this is a very simplified analysis of a space mission cost. Nevertheless, it allows us to the reach a simple conclusion: *cost for launching a satellite to low orbit is essentially independent of the planet of origin*. In consequence, it would be neither much easier nor more difficult for an extraterrestrial civilization to put its first satellite in orbit, than it was for us. And keep in mind, that while the launch cost increases with the size of the planet, planet resources also increase even faster (surface$\propto M^{2/3}$). Relative cost would then drop on a larger planet.

Once it has reached the low planetary orbit, the next step for a spacefaring civilization is to move between planets of its own stellar system. Planetary escape velocity is only ~2 times larger than initial launch cost. However, others aspects of the interplanetary exploration scale differently. Indeed, for travel between planets, velocity changes are proportional to the planetary orbital velocity, which scales as:

$$\Delta V \propto \sqrt{\frac{M_*}{d}}$$

where M_* is the mass of the star and *d* is the planet's orbital distance. We can relate those quantities by imposing the requirement that the planet lies within the habitable zone. To keep the surface temperature roughly constant, a planet circling around a brighter star must be farther from the star, while a planet circling around a fainter star must be nearer to it. This implies that $d \propto \sqrt{L_*}$. We also know the relationship between the mass of a star (M_*) and its luminosity (L_*) (Zeilik, Gregory, and Smith 1992). For a star with $M_* > 0.43M_\odot$, it follows the relationship $L \propto M^4$, and for a star with $M_* < 0.43M_\odot$, it follows the relation $L \propto 0.23\mathrm{M}^{2.3}$.

Combining these relations with the formula for orbital velocity, we can derive the following results: For $M_* > 0.43M_*$ $v_o \propto \sqrt{\frac{M_*}{\sqrt{M_*^4}}} \propto \mathrm{M}^{-1/2}$ and for $M_* < 0.43M_*$. $v_o \propto \sqrt{\frac{M_*}{0.48\mathrm{M}_*^{1.15}}} = 1.444 M_*^{-0.075}$.

Possible stellar mass range is constrained by the minimal lifetime of the star (larger than one billion years) needed to reach the stage of civilization following the formation of the planet and the need to have enough mass to generate nuclear reactions needed to heat the planet. These constraints limit the possible mass range between $0.08M_\odot$ and $1.8\ M_\odot$. Again, to avoid unneeded complexity, we do not take into account issues such as synchronous rotation, uv radiation, and atmospheric erosion that could restrict this mass range further.

Using the derived velocities from this mass range and introducing them into the rocket equation, we can calculate the cost of interplanetary orbital transfer. For the heaviest star, it is only 77.5 percent of ours and 2.1 times costlier for the lightest star. Again, relative cost is restricted to a relatively narrow range. In addition, it should be pointed out that the extent of this range is largely driven by the smallest stars, the red dwarfs. For example, for a star of $0.43\ M_\odot$ the cost increases only by 77 percent relative to the Earth's. Nevertheless, with red dwarfs being the most abundant stars, we cannot simply dismiss them from our analysis.

3.0. Adding Cost

The relative total mission cost is the sum of spacecraft cost, low orbit launch cost, and interplanetary flight cost. Today on Earth, spacecraft cost is roughly five times the low orbit launch cost (Wertz and Larson 1999). As previously, we argue that since aerospace technologies have performances very close to the limits imposed by basic physics, this ratio is relatively constant for any civilization.

From the required ΔV, we can estimate the relative cost between low earth orbit and interplanetary flight. As a benchmark, we use the Earth-Mars orbital ratio, which is likely to be the same for the successive planet in any hierarchical planetary system. We also include the deorbit burn ΔV for the target planet, which is set to be equal to the escape velocity of the planet of origin. Adding those factors, we have derived for the total ΔV:

$$\Delta V_{tot} = 1.82 v_{LEO} + 0.2 \mathrm{v}_{Earth} .$$

The total cost induced by velocity changes is calculated as usual with the help of the rocket equation using the appropriate scaling function described earlier: f_{Leo} for low orbit launch and f_{int} for interplanetary travel. For consistency, we also introduce a scaling cost factor between 1.82 ΔV_{LEO} (14.4 km/s) and 0.2 ΔV_{Earth} (5.95 km/s). The scaling factor is equal to 0.089. The total cost equation is then equal to:

$$\mathrm{cost}_{total} = \mathrm{cost}_{spacecraft} + f_{LEO}\mathrm{cost}_{LEO} + f_{inter}\mathrm{cost}_{inter} ,$$

which reduces to:

$$\mathrm{cost}_{total} = 5\left(f_{LEO}\,\mathrm{cost}_{LEO}\right) + f_{LEO}\,\mathrm{cost}_{LEO} + 0.089\, f_{inter}\,\mathrm{cost}_{LEO}$$

and finally to:

$$\boxed{\mathrm{cost}_{total} = \left(6 f_{LEO} + 0.089 f_{inter}\right)\mathrm{cost}_{LEO}}.$$

In consequence, the relative cost of space exploration, in the best case, for a star of 1.8 $M_{\odot}$ and a planet of 0.3 $M_{\oplus}$, would be 28 percent cheaper. In the worst case, $M_{\square} = 0.08M_{\odot}$ and a planet of three $M_{\oplus}$, would be only 56 percent more expensive.

It should be noted than the scaling cost factor between the low orbit velocity change and the interplanetary velocity change is a key factor in this discussion. Another orbital scenario could shift the sensitivity from planetary parameter to stellar parameter. For example, if we calculate the cost of sending a probe directly to interstellar space from a planetary surface, the ΔV needed is given by:

$$\Delta v_{tot} = 0.41 v_{LEO} + 0.41 v_{Earth} \ .$$

The scaling cost factor between the $0.41\Delta V_{LEO}$ (3.28 km/s) and 0.41 ΔV_{Earth} (12.33 km/s) is equal to 42.9. Using the same approach as previously, we derived:

$$\boxed{\text{cost}_{total} = \left(6 f_{LEO} + 42.9 f_{inter}\right)\text{cost}_{LEO}}.$$

In the worst case (big planet and small star), the relative space mission cost would be twice as expensive as on Earth. However, if we restrict the planet of origin to stars with mass larger than 0.43 $M_{\oplus}$, worst case would be only 71 percent more expensive. In the best case (small planet and big star) the cost drops by 23 percent. As we can see, cost change is minimal for a large parameter space.

This analysis also yields an interesting observation. Since our Sun is a relatively big star and hence is relatively rare, our interplanetary exploration cost is among the cheapest possible! This might have provided a head start to our space program, compared to extraterrestrial civilizations.

4.0. Conclusion

Even if this analysis is rather crude, it can provide some insight into the relative cost of space exploration for extraterrestrial civilizations. First, for realistic orbital and interplanetary missions, the relative cost range is rather small (<2) even in a worst case analysis. Nevertheless, this analysis intentionally excluded other factors, such as radiation level in space, that might play an important role. It is also possible that a civilization inhabits a moon of a larger planet. This configuration will certainly increase the difficulty significantly, but, in our opinion, not to the point of making space exploration impossible.

As a general conclusion, the planet and star of origin of an extraterrestrial civilization are likely to have little importance in the development of its indigenous space exploration. In consequence, if there is a physical restriction to interplanetary space travel, it is not caused by the local gravitational potential well but by other factors not examined here.

In addition, failure to initiate a successful space program due to its prohibitive cost cannot be invoked as an explanation for the Fermi paradox. Communication by physical artifacts cannot be dismissed on this basis either. The same can be said about SETI enabling technology that needs to be implemented in space, which also cannot be dismissed for this reason.

Works Cited

Boyce, C. 1979. *Extraterrestrial encounter: A personal perspective.* New York: Chartwell Books.

Bracewell, R. N. 1960. Communications from superior galactic communities. *Nature* 186: 670–71.

Heidmann, J. 1994. Saha Crater: A candidate for a SETI lunar base. *Acta Astronautica* 32: 471–72.

Maccone, C. 2011. Interstellar radio links enabled by gravitational lenses of the Sun and stars. In *Communication with extraterrestrial intelligence (CETI)*, ed. D. A. Vakoch. Albany, NY: State University of New York Press.

Rose, C., and G. Wright. 2004. Inscribed matter as an energy efficient means of communication with an extraterrestrial civilization. *Nature* 431 :47–49.

Seager, S., M. Kuchner, C. A. Hier-Majumder, and B. Militzer. 2007. Mass-radius relationship for solid exoplanets. *Astrophysical Journal* 669 (2): 1279–97.

Swift, D. C., J. Eggert, D. G. Hicks, S. Hamel, K. Caspersen, E. Schwegler, G. W. Collins, and G. J. Ackland. 2010. Mass-radius relationships for exoplanets: http://arxiv.org/pdf/1001.4851.

Tsiolkovsky, K. E. 1903. The exploration of cosmic space by means of reaction devices (Исследование мировых пространств реактивными приборами). *The Science Review* (5).

von Neumann, J., and A. W. Burks. 1966. *Theory of self-reproducing automata*. Urbana: Univ. of Illinois Press.

Wertz, J. R., and W. J. Larson. 1999. *Space mission analysis and design*, 3rd ed. El Segundo, CA: Microcosm Press.

Zeilik, M., S. A. Gregory, and E. V. Smith. 1992. *Introductory astronomy and astrophysics*, 3rd ed. Fort Worth: Saunders College Publishing.